Problem 1

设存在元素y使得x∨y=1且x∧y=0, 则

y = y∧1 = y∧(x∨x¯) = (y∧x)∨(y∧x¯) = (x∧y)∨(y∧x¯) = 0∨(y∧x¯) =

y∧x¯ = (x¯∧y)∨0 = (x¯∧y)∨(x∧x¯) = x¯∧(y∨x) = x¯∧(x∨y) = x¯∧1 = x¯

存在唯一的元素y= x¯使得x∨y=x∨x¯=1, x∧y=x∧x¯=0

Problem 2

a) 成立, 列出真值表可证

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| x y z | 0 0 0 | 0 0 1 | 0 1 0 | 0 1 1 | 1 0 0 | 1 0 1 | 1 1 0 | 1 1 1 |
| x⊕(y⊕z) | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 |
| (x⊕y)⊕z | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |

b) 不成立, 取x=1, y=1, z=1, 1+(1⊕1)=1+0=1, (1+1)⊕(1+1)=1⊕1=0

c) 不成立, 取x=1, y=1, z=0, 1⊕(1+0)=1⊕1=0, (1⊕1)+(1⊕0)=0+1=1

Problem 3

a≤b ⇔ a∧b=a, 则a∧b’ = (a∧b)∧b’ = a∧(b∧b’) = a∧0 = 0

a∧b’=0 ⇔ a’∨b = (a∧b’)’ = 1

a’∨b=1 ⇔ a = a∧1 = a∧(a’∨b) = (a∧a’)∨(a∧b) = 0∨(a∧b) = a∧b) ⇔ a≤b

Problem 4

易见⊕运算在B上封闭, 又对任意x, y, z∈B, 有

(x⊕y)⊕z = (((x∧y′)∨(x′∧y))∧z’)∨(((x∧y′)∨(x′∧y))’∧z)

x⊕(y⊕z) = (x∧((y∧z’)∨(y’∧z))’)∨(x’∧((y∧z’)∨(y’∧z)))

则(x⊕y)⊕z = x⊕(y⊕z) = (x∧y∧z)∨(x∧y’∧z’), ⊕满足结合性

又对任意x∈B有x⊕0 = 0⊕x = 0, 则0是单位元

对任意x∈B有x⊕x = x⊕x = 0, 任意x是自身的逆元, <B, ⊕>构成群

Problem 5

对任意a, b, c∈B, 若a≤c则有a∨c=c, a∨(b∧c) = (a∨b)∧(a∨c) = (a∨b)∧c

Problem 6

运用数学归纳法, 当n=2时(a1∨a2)’=a1’∧a2’, (a1∧a2)’=a1’∨a2’, 即德摩根律

假设对于n=k命题成立, 则对n=k+1有

1) (a1∨a2∨···∨ak+1)′ = ((a1∨a2∨…∨ak)∨ak+1)’ =

(a1’∧a2’∧···∧ak’)∧ak+1’ = a1’∧a2’∧···∧ak’∧ak+1’

2) (a1∧a2∧···∧ak+1)′ = ((a1∧a2∧…∧ak)∧ak+1)’ =

(a1’∨a2’∨···∨ak’)∨ak+1’ = a1’∨a2’∨···∨ak’∨ak+1’

命题对n=k+1也成立, 由数学归纳法n对全部n∈N且n≥2恒成立, 证毕

Problem 7

对任意a, b∈B, 若a≤b, 有a∧b=a, a’∨b’ = (a∧b)’ = a’, 则b’≤a’

反之若b’≤a’, 有a’∧b’ = b’, a∨b = (a’∧b’)’ = (b’)’ = b, 即a≤b

Problem 8

任一有限布尔代数B同构于B中所有的原子构成集合A的幂集代数系统P(A)

则两个有限布尔代数同构的充分必要条件是元素个数相同

已知B1B2,B2B3, 则|B1| = |B2|, |B1| = |B3|, 则B2B3

Problem 9

1) 0-7之间的斐波那契数有1, 2, 3, 5, 则F的真值表为

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| x y z | 0 0 0 | 0 0 1 | 0 1 0 | 0 1 1 | 1 0 0 | 1 0 1 | 1 1 0 | 1 1 1 |
| F | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 |

2) 由真值表可得F = x¯y¯z + x¯yz¯ + x¯yz + xy¯z

3) 根据真值表作卡诺图如下, 则有F = x¯y + y¯z

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | yz | yz¯ | y¯z¯ | y¯z |
| **x** |  |  |  | 1 |
| **x¯** | 1 | 1 |  | 1 |